

Single pickup and delivery model with partial satisfaction

1 Mathematical Model

Sets:

V : Set of all locations (including depot 0).

Parameters:

c_{ij} : Travel cost (or distance) from station i to j .

S_i : Pickup demand at station i

D_i : Delivery demand at station i

Q : Vehicle capacity.

P_i^s : Profit for meeting each unit of surplus demand of station i .

P_i^d : Profit for meeting each unit of deficit demand of station i .

Decision Variables:

$x_{ij} \in \{0, 1\}$ 1 if the vehicle travels from i to j , else 0.

s_i The amount of met pickup demand of station i

d_i The amount of met delivery demand of station i

$l_i \geq 0$ Load of the vehicle after visiting station i .

$l_0 \geq 0$ Initial load at the depot (decision variable).

u_i Visit order of station i

Objective Function:

$$\max \sum_{i \in V} (P_i^s s_i + P_i^d d_i) - \sum_{i \in V} \sum_{j \in V} c_{ij} x_{ij} \quad (1)$$

Constraints:

1. Vehicle Leaves and Returns to Depot

$$\sum_{j \in V} x_{0j} = 1 \quad (2)$$

$$\sum_{i \in V} x_{i0} = 1 \quad (3)$$

2. Flow Conservation (Continuity)

$$\sum_{j \in V} x_{ij} \leq 1, \quad \forall i \in V \setminus \{0\} \quad (4)$$

$$\sum_{j \in V} x_{ij} = \sum_{j \in V} x_{ji}, \quad \forall i \in V \setminus \{0\} \quad (5)$$

$$s_i \leq M \sum_j x_{ij}, \quad \forall i \in V \setminus \{0\} \quad (6)$$

$$d_i \leq M \sum_j x_{ij}, \quad \forall i \in V \setminus \{0\} \quad (7)$$

3. Load Balance for Pickups and Deliveries

$$l_j \geq l_i + s_j - d_j - M(1 - x_{ij}), \quad \forall i, j \in V, i \neq j \quad (8)$$

$$l_j \leq l_i + s_j - d_j + M(1 - x_{ij}), \quad \forall i, j \in V, i \neq j \quad (9)$$

4. Load Capacity Constraints

$$0 \leq l_i \leq Q, \quad \forall i \in V \quad (10)$$

5. surplus and deficit demand boundaries

$$0 \leq s_i \leq S_i \quad (11)$$

$$0 \leq d_i \leq D_i \quad (12)$$

6. Subtour Elimination Constraints (MTZ Formulation)

$$u_j \geq u_i + 1 - (|V| - 1)(1 - x_{ij}), \quad \forall i, j \in V, i \neq j, i \neq 0, j \neq 0 \quad (13)$$

$$1 \leq u_i \leq |V| - 1, \quad \forall i \in V, i \neq 0 \quad (14)$$